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Institute of Mathematics, University of Debrecen
under the auspices of
the Working Group on Generalized Convexity

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General Information

The conference is held in Hotel Aurum in Hajdúszoboszló, Hungary, from Sunday, August 27 (arrival day) to Saturday, September 2, 2017 (departure day). The participation fee covers full board, accommodation (in double rooms), the use of the spa and the sauna in Hotel Aurum, registration materials, the fees of the excursion, the banquet and other social events of the meeting.

All conference *talks* are given in the *Main Lecture Room*, which is equipped with a computer, a data projector, and a paper-board. The Main Lecture Room and the Coffee Breaks are in the In Hotel.

The duration of a regular talk and a plenary talk is at most 20 minutes and 45 minutes respectively, which is followed by a discussion of at most 5 minutes. There are no breaks between the talks within a session, therefore the schedule of the individual talks is only approximative. Speakers cannot inherit time from the previous talk.

During the conference internet is available in the hotel with a wireless connection.

You can find the program and the abstracts in this booklet. Your questions may help the Organizing Committee to improve organization, so do not hesitate to contact us. We hope that our conference will be interesting and successful and you will enjoy your stay in Hajdúszoboszló.

XII International Symposium
on Generalized Convexity and Monotonicity
August 27 - September 2, 2017, Hajdúszoboszló, Hungary

Program

	Monday		Tuesday
07:00–08:50	Breakfast	07:00–09:00	Breakfast
08:50–09:00	Opening		
09:00–10:40	1st Morning Session	09:00–10:40	1st Morning Session
10:40–11:10	Coffee Break	10:40–11:10	Coffee Break
11:10–12:25	2nd Morning Session	11:10–12:25	2nd Morning Session
12:30–14:00	Lunch	12:30–14:00	Lunch
15:00–16:40	1st Afternoon Session	15:00–16:40	1st Afternoon Session
16:40–17:10	Coffee Break	16:40–17:10	Coffee Break
17:10–18:25	2nd Afternoon Session	17:10–18:25	2nd Afternoon Session
18:30–20:00	Dinner	18:30–20:00	Dinner

	Wednesday		Thursday
07:00–09:00	Breakfast	07:00–09:00	Breakfast
09:00–10:40	1st Morning Session	09:00–10:40	1st Morning Session
10:40–11:10	Coffee Break	10:40–11:10	Coffee Break
11:10–12:00	2nd Morning Session	11:10–12:25	2nd Morning Session
12:00–13:00	Lunch	12:30–14:00	Lunch
13:00–22:00	Excursion to Lake Tisza	15:00–16:40	1st Afternoon Session
	Ecocenter in Poroszló	16:40–17:10	Coffee Break
	with dinner	17:10–18:40	General Assembly of WGGC
	at the Tavern of Hortobágy	19:30–	Festive Dinner

	Friday		Saturday
07:00–09:00	Breakfast	07:00–09:00	Breakfast
09:00–10:40	1st Morning Session		
10:40–11:10	Coffee Break		
11:10–12:25	2nd Morning Session		
12:30–14:00	Lunch		
15:00–16:10	1st Afternoon Session		
18:00–	Farewell Party		

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List of Talks

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2. Masoud Ahookhosh, *Finding zeros of mappings satisfying Hölder metric subregularity*,
3. Mohammad Hossein Alizadeh, *Fitzpatrick function for premonotone operators*,
4. Roman Badora, *On monotonic functions between lattices*,
5. Mihály Bessenyei, *The affine separation problem revisited*,
6. Zoltán Boros, *Monotonicity of the $\mathbb{Q}$-subdifferential of Jensen-convex functions*,
7. Riccardo Cambini, *Generating the efficient frontier of a class of bicriteria generalized fractional programs*,
8. Jacek Chudziak, *On convexity of the zero utility principle*,
9. Indira Debnath, *Duality gaps for I-fuzzy linear programming problems under exponential membership function*,
10. Maria Bernadette Donato, *Variational formulation of a general equilibrium model with incomplete financial markets and numeraire assets: existence*,
11. Francisco Facchinei, *Asynchronous parallel algorithms for the minimization of nonsmooth, nonconvex functions*,
12. Włodzimierz Fechner, *Some remarks on Hlawka's inequality*,
13. Fabián Flores Bazán, *The standard quadratic optimization problem and its extensions: strong duality, hidden convexity and S-lemma*,
14. Roman Ger, *Higher order Jensen-convex functionals that are measurable on curves*,
15. Attila Gilányi, *On convex functions of higher order with a modulus*,
16. Dorota Głazowska, *Convex means*,
17. Anca Grad, *On a particular monotone inclusion by means of penalty schemes with inertial effects*,
18. Rachana Gupta, *Gap functions for quasi equilibrium problems*,
19. Shiv Kumar Gupta, *On multiobjective interval valued fractional optimization problems*,
20. Nicolas Hadjisavvas, *Quasiconvex functions, adjusted sublevel sets, and global optimization*,
21. René Henrion, *Probabilistic constraints: convexity issues and beyond*,
22. László Horváth, *Cyclic refinements of the different versions of operator Jensen's inequality*,
23. Lidia Huerga, *Characterization of proper efficiency in multiobjective optimization through nonlinear scalarization*,
24. Alfredo N. Iusem, *Second order asymptotic functions with applications to Quadratic Programming*,
25. Justyna Jarczyk, *Convex means*,
26. Witold Jarczyk, *Convexity in abelian semigroups*,
27. Anurag Jayswal, *Modified objective function approach for multitime variational problems*,

28. Alireza Kabgani, *A nonsmooth semi-infinite multiobjective optimization problem using convexificators*,
29. Tibor Kiss, *Implications among generalized convexity properties of real functions*,
30. Fumiaki Kohsaka, *Approximating minimizers of convex functions in geodesic metric spaces with curvature bounded above*,
31. Dawid Kotrys, *Separation by Jensen and affine stochastic processes*,
32. Alexandru Kristály, *Convexity vs. curvature*,
33. Felipe Lara, *Asymptotic analysis for quasiconvex functions with applications*,
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37. Gyula Maksa, *Results related to the invariance equation* $K\circ(M,N)=K$,
38. Monica Milasi, *Solutions of quasi-variational inequalities through variational inequalities*,
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41. Noémi Nagy, *Separation theorems for approximately convex functions*,
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43. Andrzej Olbryś, *On* T*-Schur-convex functions* ,
44. Zsolt Páles, *Support theorems in an abstract setting*,
45. Cornel-Sebastian Pintea, *Generalized monotone operators on finite dimensional spaces. Applications*,
46. Teresa Rajba, *On strongly quasi convex functions*,
47. Juan Luis Rodenas-Pedregosa, *Nonlinear scalarization in real linear spaces and vector equilibrium problems*,
48. Vivek SINGH, *Second order* (H_p,r)*-invexity and duality for minimax fractional programming with square root term*,
49. Éva Székelyné Radácsi, *Characterization of convexity with respect to smooth Chebyshev systems*,
50. Patricia Szokol, *Transformations preserving norms of means of positive operators*,
51. Tomasz Szostok, *Some applications of Levin-Stechkin theorem and it's generalizations*,
52. Flemming Topsøe, *Models of information and global optimization without Lagrange multipliers*,
53. Csaba Varga, *A geometric characterization of Hardy inequality in Minkowski space*,
54. Csaba Vincze, *Lazy orbits: an optimization problem on the sphere*,
55. Peter Volkmann, *Quasimonotonicity and functional inequalities*,
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57. Yuri Zelinskii, *Linearly convex sets in complex analysis*,

Abstracts

Mirosław Adamek
(Department of Mathematics, University of Bielsko-Biała)

Remarks on F-convex functions

In this speech we discuss some counterparts of classical results for a new class of functions, namely F-midconvex functions. We obtain Bernstein-Doetsch, Ostrowski and Sirpiński type theorems. We also present Kuhn type result and Jensen-type inequality. In particular, these results generalize the results presented in the paper: A. Azócar, J. Giménez, K. Nikodem and Sánchez, *On strongly midconvex functions*, Opuscula Math. **31** (2011), 15–26.

Masoud Ahookhosh

(Luxembourg Centre for Systems Biomedicine, University of Luxembourg)

Finding zeros of mappings satisfying Hölder metric subregularity

(joint work with Francisco J. Aragón Artacho, Ronan M.T. Fleming, Vuong Phan)

We aim to find zeros of mappings satisfying Hölder metric subregularity with Levenberg-Marquardt methods. To do so, we first propose an adaptive formula for the Levenberg-Marquardt parameter and analyse the local convergence of the method under Hölder metric subregularity. We then introduce a bounded version of the Levenberg-Marquardt parameter and analyse the local convergence of the modified method under the Lojasiewicz gradient inequality. We finally report encouraging numerical results confirming the theoretical findings for the problem of computing moiety conserved steady states in biochemical reaction networks. This problem can be cast as finding a zeros of a mapping, which satisfies the Hölder metric subregularity.

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Mohammad Hossein Alizadeh
(Institute for Advanced Studies in Basic Sciences, IASBS)

Fitzpatrick function for pre-monotone operators

Suppose that X is a real Banach space, with topological dual space X^*. Given an operator $T : X \to 2^{X^*}$ with domain $D(T) = \{x \in X : T(x) \neq \emptyset\}$, with range

$$R(T) = \{x^* \in X^* : \exists\, x \in X \text{ s.t. } x^* \in T(x)\}$$

and with graph $\operatorname{gr} T := \{(x, x^*) \in X \times X^* : x^* \in T(x)\}$.

Recall that T is said to be monotone if for every $x, y \in D(T)$, $x^* \in T(x)$ and $y^* \in T(y)$,

$$\langle x^* - y^*,\ x - y\rangle \geq 0,$$

and T is said to be maximal monotone if its graph is not properly included in any other monotone graph.

The Fitzpatrick function of a monotone operator was introduced in [3] by Fitzpatrick. Around 13 years later, a convex representation (Fitzpatrick function) of maximal monotone operators was rediscovered by Martinez-Legaz and Théra [4]. The notion of pre-monotone operators for the finite-dimensional case is introduced in [2]. In Banach spaces, some basic properties of pre-monotone operators are studied in [1].

Definition. (i) Given an operator $T : X \to 2^{X^*}$ and a map $\sigma : D(T) \to \mathbb{R}_+$, T is said to be *σ-monotone* if for every $x, y \in D(T)$, $x^* \in T(x)$ and $y^* \in T(y)$,

$$\langle x^* - y^*,\ x - y\rangle \geq -\min\{\sigma(x), \sigma(y)\}\|x - y\|. \tag{1}$$

(ii) An operator T is called *pre-monotone* if it is *σ-monotone* for some $\sigma : D(T) \to \mathbb{R}_+$.

(iii) A σ-monotone operator T is called *maximal σ-monotone*, if for every operator T' which is σ'-monotone with $\operatorname{gr} T \subseteq \operatorname{gr} T'$ and σ' an extension of σ, one has $T = T'$.

As for monotone case, each σ-monotone operator has a maximal σ-monotone extension [1].

We define the Fitzpatrick function for σ-monotone operators like as the monotone case:

Definition. Let X be a Banach space and $T : X \to 2^{X^*}$ be a σ-monotone operator. The *Fitzpatrick function* associated with T is the function $\mathcal{F}_T : X \times X^* \to \mathbb{R} \cup \{+\infty\}$ defined by

$$\mathcal{F}_T(x, x^*) = \sup_{(y,y^*) \in \operatorname{gr} T} \left(\langle x^*, y\rangle + \langle y^*, x\rangle - \langle y^*, y\rangle\right)$$

which is norm$\times$weak*- lsc and convex on $X \times X^*$.

We need the following lemma for proving Theorem 1.

Lemma 1. *Suppose that T is maximal σ-monotone for some $\sigma : D(T) \to \mathbb{R}_+$. Then for all $(x, x^*) \in X \times X^*$ we have*

$$\inf_{(y,y^*)\in \operatorname{gr} T} \langle y^* - x^*, y - x\rangle \leq 0. \tag{2}$$

Theorem 1. *Suppose that $T : X \to 2^{X^*}$ is a maximal σ-monotone operator. Then*

$$\mathcal{F}_T(x, x^*) \geq \langle x^*, x\rangle \qquad \forall (x, x^*) \in X \times X^*. \tag{3}$$

If $(x, x^*) \notin \operatorname{gr} T$, then we have strict inequality in (3). If $(x, x^*) \in \operatorname{gr} T$ and $\sigma(x) = 0$, then the equality holds.

The following proposition shows that if the equality holds in Theorem 1 for all $x^* \in T(x)$, then $\sigma_T(x) = 0$.

Proposition 1. *Suppose that $T : X \to 2^{X^*}$ is σ-monotone. Let $x \in X$ and for each $x^* \in T(x)$, $\mathcal{F}_T(x, x^*) = \langle x^*, x\rangle$, then $\sigma_T(x) = 0$.*

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[1] Alizadeh, M. H., Hadjisavvas, N., Roohi, M.: *Local boundedness properties for generalized monotone operators*, J. Convex Anal. **19** (2012) 49–61.

[2] Iusem, A. N., Kassay G., Sosa, W.: *An existence result for equilibrium problems with some surjectivity consequences*, J. Convex Anal. **16** (2009) 807–826.

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[4] Martinez-Legaz, J.-E., Thera, M.: *A convex representation of maximal monotone operators*, J. Nonlinear Convex Anal., **2(2)** (2001) 243–247.

Roman Badora
(Institute of Mathematics, University of Silesia)

On monotonic functions between lattices

(joint work with Tomasz Kochanek, Barbara Przebieracz)

In the talk we present separation theorems (or sandwich type results) for monotonic functions between lattices. As a consequence, a solution of Ulam's type stability problem for monotonic functions is obtained.

Mihály Bessenyei
(Instituite of Mathematics, University of Debrecen)

The affine separation problem revisited

The aim of this talk is to characterize in terms of inequalities those pairs of real functions (acting on a convex subset of a vector space) that possess an affine separator. The main result is originally due to Behrends and Nikodem. Their method is based on the Hahn–Banach Theorem and a variant of the Helly Theorem. In our approach, a direct and independent proof is presented via the Radon and the Helly Theorems.

Zoltán Boros

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Monotonicity of the $\mathbb{Q}$-subdifferential of Jensen-convex functions

(joint work with Zsolt Páles)

Let $\mathbb{F}$ denote a subfield of the real number field $\mathbb{R}$ and X be a vector space over $\mathbb{F}$. Let $\mathcal{A}_{\mathbb{F}}$ denote the set of all $\mathbb{F}$-linear mappings $A : X \to \mathbb{R}$, and let $\mathcal{P}_0(\mathcal{A}_{\mathbb{F}})$ denote the family of all non-empty subsets of $\mathcal{A}_{\mathbb{F}}$. Let D be a non-void, $\mathbb{F}$-algebraically open, $\mathbb{F}$-convex subset of X.

Let $f : D \to \mathbb{R}$ and $x_0 \in D$. The set

$$\partial_{\mathbb{F}} f(x_0) = \{\, A \in \mathcal{A}_{\mathbb{F}} \mid f(x_0) + A(x - x_0) \le f(x) \text{ for every } x \in D \,\}$$

is called the $\mathbb{F}$-*subdifferential of* f *at* x_0.

Following the ideas of Minty [2] and Rockafellar [3, 4], we show that the $\mathbb{F}$-subdifferential $\partial_{\mathbb{F}} f : D \to \mathcal{P}_0(\mathcal{A}_{\mathbb{F}})$ of an $\mathbb{F}$-convex function f fulfils $\sum_{j=0}^{n} A_j(x_{j+1} - x_j) \le 0$ for every $n \in \mathbb{N}$, $x_j \in D$ $(j = 0, 1, \ldots, n, n+1)$ with $x_{n+1} = x_0$, and $A_j \in \partial_{\mathbb{F}} f(x_j)$ $(j = 0, 1, \ldots, n)$. We characterize $\mathbb{F}$-subdifferentials of $\mathbb{F}$-convex functions as maximal mappings from D into $\mathcal{P}_0(\mathcal{A}_{\mathbb{F}})$ with this property [1]. Various concepts of monotonicity and maximality are considered and compared.

References

[1] Z. Boros and Zs. Páles, $\mathbb{Q}$-*subdifferential of Jensen-convex functions*, J. Math. Anal. Appl. **321** (2006), 99–113.
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Riccardo Cambini
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Generating the efficient frontier of a class of bicriteria generalized fractional programs

(joint work with Laura Carosi)

A particular class of bicriteria maximization problems over a compact polyhedron is considered. The first component of the objective function is the ratio of powers of affine functions and the second one is linear.

Several theoretical properties are provided, such as the pseudoconcavity of the first criterium of the objective function, the connectedness and compactness of both the efficient frontier and the set of efficient points.

The obtained results allows us to propose a new simplex-like solution method for generating the whole efficient frontier. To better clarify the use of the suggested algorithm several examples are described and the results of a computational test are presented.

Jacek Chudziak

(Faculty of Mathematics and Natural Sciences, University of Rzeszów)

On convexity of the zero utility principle

Assume that $\mathcal{X}_+$ is a family of all non-negative bounded random variables on a given probability space. Consider an insurance company, covering the risks represented by the elements of $\mathcal{X}_+$. If $u : \mathbb{R} \to \mathbb{R}$ is its continuous and strictly increasing utility function then for every $X \in \mathcal{X}_+$ there exists a unique real number $H_u(X)$ such that

$$E[u(H_u(X) - X)] = 0. \tag{1}$$

Therefore, equation (1) defines in an implicit way a functional H_u on $\mathcal{X}_+$, called the zero utility principle. In the talk we deal with the properties of this functional. In particular, some aspects of convexity of H_u are discussed.

Indira P. Debnath
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Duality gaps for I-fuzzy linear programming problems under exponential membership function

(joint work with S. K. Gupta)

Fuzziness is ever presented in real life decision making problems. In this paper, we adapt the pessimistic approach to study a pair of linear primal-dual problem under intuitionistic fuzzy (I-fuzzy) environment.

The I-fuzzy versions of the classical primal-dual pair of linear programming problem is

$$\begin{aligned} \text{(IFLP)} \quad & \text{Find } x \in \mathbb{R}^n, \text{ such that} \\ & c^T x \gtrsim_{IF} z_0, \\ & Ax \lesssim_{IF} b, \\ & x \geq 0, \end{aligned}$$

and

$$\begin{aligned} \text{(IFLD)} \quad & \text{Find } w \in \mathbb{R}^m, \text{ such that} \\ & b^T w \lesssim_{IF} w_0, \\ & A^T x \gtrsim_{IF} c, \\ & w \geq 0. \end{aligned}$$

To indicate the decision maker's level of satisfaction and dissatisfaction in the pessimistic approach for the objective function and the system of constraints, we hereby consider the following form of exponential membership and non-membership function:

$$\mu_0(c^T x) = \begin{cases} 1 & \text{if } c^T x \geq z_0, \\ \dfrac{e^{\alpha_0((c^T x - z_0)/p_0)} - e^{-\alpha_0}}{1 - e^{-\alpha_0}} & \text{if } z_0 - p_0 \leq c^T x < z_0, \\ 0 & \text{if } c^T x < z_0 - p_0, \end{cases}$$

$$\nu_0(c^Tx) = \begin{cases} 1 & \text{if } c^Tx \leq z_0 - p_0, \\ \dfrac{e^{\alpha_0((z_0-p_0-c^Tx)/q_0)} - e^{-\alpha_0}}{1-e^{-\alpha_0}} & \text{if } z_0 - p_0 < c^Tx \leq z_0 - p_0 + q_0, \\ 0 & \text{if } c^Tx > z_0 - p_0 + q_0, \end{cases}$$

and for $i = 1, 2, ..., m$,

$$\mu_i(A_ix) = \begin{cases} 1 & \text{if } A_ix \leq b_i, \\ \dfrac{e^{\alpha_i((b_i-A_ix)/p_i)} - e^{-\alpha_i}}{1-e^{-\alpha_i}} & \text{if } b_i < A_ix \leq b_i + p_i, \\ 0 & \text{if } A_ix > b_i + p_i, \end{cases}$$

$$\nu_i(A_ix) = \begin{cases} 1 & \text{if } A_ix \geq b_i + p_i, \\ \dfrac{e^{\alpha_i((A_ix-b_i-p_i)/q_i)} - e^{-\alpha_i}}{1-e^{-\alpha_i}} & \text{if } b_i + p_i - q_i \leq A_ix < b_i + p_i, \\ 0 & \text{if } A_ix < b_i + p_i - q_i. \end{cases}$$

Following Angelov's principle, we converted the above formulation of the I-fuzzy linear models to the crisp optimization problems. With this and using the exponential membership and non-membership functions, we generate the duality results to represent the decision maker's satisfaction and dissatisfaction level. Further, we have compared the duality gaps obtained for the I-fuzzy dual pair, using exponential and linear membership function and shown that the results with exponential are better than the linear membership function, since it reduces the duality gaps significantly and hence more close to practical applicability of the problem. A numerical example for I-fuzzy linear problem has also been formulated and the duality gaps have been discussed.

Maria Bernadette Donato
(Department of Mathematics and Computer Science, University of Messina)

Variational formulation of a general equilibrium model with incomplete financial markets and numeraire assets: existence

We present a general equilibrium model with incomplete financial markets and numeraire assets. We assume that there are 2 periods of time, say today and tomorrow. In period 0, households exchange goods and assets and then consumption takes place; in period 1, one of S possible states of nature occurs. In each of them, assets pay their returns which are measured in units of a given physical good, i.e., the numeraire commodity; households exchange goods; finally, consumption takes place. We define a consumption, portfolio holding, commodity and asset price vector as an equilibrium vector associated with a given economy if at those prices and economies households maximize, and market clears. While the existence proof by Geneakoplos and Polemarchakis (1986) uses a fixed point argument, we provide an existence result in terms of variational inequalities. More precisely, we reformulate the problem by means of a quasi-variational inequality involving set-valued maps, under quasi-concavity and continuity assumptions on utility functions. Finally, by applying the variational inequality theory, we investigate on the existence of equilibrium points. That approach allows to get the desired result under some different and more realistic assumptions than those usually made in the literature.

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Francisco Facchinei
(Department of Computer, Control, and Management Engineering,
University of Rome La Sapienza)

Asynchronous parallel algorithms for the minimization of nonsmooth, nonconvex functions

(joint work with Loris Cannelli, Vyacheslav Kungurtsev, Gesualdo Scutari)

We present recent advancements in the field of asynchronous parallel methods for the minimization of the sum of a differentiable function and a possibly nonsmooth, convex regularizer subject to constraints. In recent years, problems of this kind have played an important role in many applicative fields and instances of ever increasing dimensions need to be solved. Asynchronous methods play a key role in the solution of these large problems. After reviewing recent results in the field, we discuss, in particular, a rather general framework along with its main convergence properties and report numerical results.

Włodzimierz Fechner
(Institute of Mathematics, Łódź University of Technology)

Some remarks on Hlawka's inequality

Assume that X is a normed linear space and $x, y, z \in X$. By *Hlawka's inequality* we mean

$$\|x+y\| + \|y+z\| + \|x+z\| \le \|x+y+z\| + \|x\| + \|y\| + \|z\|. \tag{1}$$

Hlawka's functional inequality is of the form:

$$f(x+y) + f(y+z) + f(x+z) \le f(x+y+z) + f(x) + f(y) + f(z). \tag{2}$$

It can be studied for a real-valued function f defined on a topological group or on the real line (see [1, 2]). During the talk some recent results related to (1) and (2) will be reported.

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Fabián Flores Bazán

(Department of Mathematical Engineering, University of Concepcion)

The standard quadratic optimization problem and its extensions: strong duality, hidden convexity and S-lemma

(joint work with Gabriel Cárcamo and Stephanie Caro)

Many formulations of quadratic allocation problems, portfolio optimization problems, the maximum weight clique problem, among others, take the form as the well-known standard quadratic optimization (StQO) problem, which consists in minimizing a homogeneous quadratic function on the usual simplex in the non negative orthant. We propose to analyze the same problem when the simplex is substituted by a convex and compact base of any pointed, closed, convex cone (so, the cone of positive semidefinite matrices or the cone of copositive matrices are particular instances). Three main duals (for which a semi-infinite formulation of the primal problem is required) are associated, and we establish some characterizations of strong duality with respect to each of the three duals in terms of copositivity of the Hessian of the quadratic objective function on suitable cones. Such a problem reveals a hidden convexity and the validity of S-lemma. In case of bidimensional quadratic optimization problems, copositivity of the Hessian of the objective function is characterized, and the case when every local solution is global. Finally, the L^2-version of the StQO problem is formulated, and the validity of its strong duality property is characterized.

Roman Ger
(Institute of Mathematics, Silesian University)

Higher order Jensen-convex functionals that are measurable on curves

I. Labuda and R. D. Mauldin [2] have solved in affirmative the following S. Mazur's problem posed about 1935 (see [3]):

> *"In a space E of type (B), there is given an additive functional $F(x)$ with the following property: If $x(t)$ is a continuous function in $0 \leq t \leq 1$ with values in E, then $F(x(t))$ is a measurable function. Is $F(x)$ continuous?"*

In [1] we have shown that the same remains true in the case where F is a Jensen-convex functional on an open and convex subdomain of a real Banach space. Now, we shall study the possibilities of an extension of this result to Jensen-convexity of higher orders.

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Attila Gilányi
(Faculty of Informatics, University of Debrecen)

On convex functions of higher order with a modulus

(joint work with Nelson Merentes, Kazimierz Nikodem and Zsolt Páles)

In order to extend and generalize our results presented at the 11$^{\text{th}}$ International Symposium on Generalized Convexity and Monotonicity, we consider $(t_1, \ldots, t_n)$-Wright-convex functions with a modulus c, i.e., real valued functions defined on an open interval I satisfying the inequality

$$\Delta_{t_1 h} \cdots \Delta_{t_n h} f(x) \geq c n! (t_1 h) \cdots (t_n h)$$

for all $x \in I$, $h > 0$ such that $x + (t_1 + \cdots + t_n)h \in I$, where n is a positive integer, c is an arbitrary and $t_1, \ldots, t_n$ are positive real numbers. We give a characterization of functions of this type via generalized derivatives and we prove that the property above is localizable. As corollaries of our results, we obtain characterization theorems for (strongly) convex, (strongly) Wright-convex and (strongly) Jensen-convex functions of higher order and we show that all of these convexity properties are localizable.

Dorota Głazowska

(Faculty of Mathematics, Computer Science and Econometrics, University of Zielona Góra)

Convex means

(joint work with Justyna Jarczyk)

We report some preliminary results of our joint work on convexity of means. At the very beginning we have focused on weighted quasi-arithmetic means studying connections between their convexity and the convexity of their generators. Some examples and counterexamples are presented as well as open problems are posed. Also connections between the convexity of an arbitrary weighted quasi-arithmetic mean generated by a function and that one generated by its inverse are studied.

Shiv Kumar Gupta
(Department of Mathematics, Indian Institute of Technology Roorkee)

On multiobjective interval valued fractional optimization problems

Interval optimization problems are closely related to the inexact data in real life engineering and economic problems. These type of problems involves bounds on the values of the coefficient of the objective function. Since the real world problems involves uncertain or imprecise data, therefore it is not necessary that the coefficients involved in the objective functions is a real number. Unlike interval programming problems, robust optimization, stochastic optimization and fuzzy optimization also deals with the uncertain data. However, it is easy to deal with the interval optimization as it does not require the assumption of probabilistic distributions as in stochastic programming or the assumption of possibilistic distributions as in fuzzy programming. Two types of interval programming problems are available, viz, interval linear programming and interval nonlinear programming. We focus on the class of nonlinear interval optimization problems.

The KKT conditions for the interval optimization problems has become an interesting topic of research in recent years. Extending the idea of convexity to LU-convexity/CW-convexity, Wu has given the Karush Kuhn Tucker conditions for the interval-valued optimization problems under LU-convexity/CW-convexity assumptions.

The paper aims at extending the KKT optimality conditions for a class of multivalued fractional interval optimization problems. The main motivation of considering functions as interval fractional functions is that the uncertainity involved in many practical problems may be in the form of ratio of two functions. In the very beginning, we propose the definitions of LU-V-invex and LS-V-invex for the considered multivalued fractional interval problem. The concept of LU/LS-Pareto optimality for the problem have also been illustrated with adequate examples. Taking into consideration the concept of LU/LS-V-invexity, the KKT optimality conditions for the multivalued fractional interval problem have been established. The theoretical developments in the paper have been explained with number of nontrivial examples in suitable places throughout the manuscript

Nicolas Hadjisavvas
(Department of Product and Systems Design Engineering, University of the Aegean)

Quasiconvex functions, adjusted sublevel sets, and global optimization

(joint work with Suliman Al-Homidan, Loai Shaalan)

Quasiconvex functions present some difficulties in global optimization, because their graph contains "flat parts", thus a local minimum is not necessarily the global minimum. Other, more restricted classes of functions such as the semistrictly quasiconvex functions, do not present this drawback. The question at the origin of the present work was: Is it possible to write any lsc quasiconvex function f as a composition $f = g \circ h$, where g is nondecreasing, and h is semistrictly quasiconvex? If such a decomposition of f is possible, then f and h will have the same points of global minimum, so in this case we may replace f by h in the search of these points.

The answer to the question above is unfortunately "no". However, we show that the answer becomes "yes" if we replace semistrict quasiconvexity by the assumption that h is quasiconvex, and for each point $x \in \mathbb{R}^n$, the lower level set and the strict lower level set corresponding to the value $f(x)$ have the same closure. This assumption is enough to guarantee that every local minimum of h is a global minimum. It is very close to a notion introduced in [1].

As a first step to this aim, we study the continuity properties of the so-called "adjusted sublevel sets" operator for quasiconvex functions. This complements the study of the normal operator to these sets, contained in previous papers.

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René Henrion

(Weierstrass Institute, Berlin)

Probabilistic constraints: convexity issues and beyond

A probabilistic constraint is an inequality of the type $\varphi(x) \geq p$, where $p \in (0, 1]$ is a probability level and

$$\varphi(x) := \mathbb{P}(g(x, \xi) \geq 0)$$

is a probability function assigning to a decision variable x the probability of satisfying a random inequality system $g(x, \xi) \geq 0$ with ξ referring to a random vector. Such constraints represent an important tool within stochastic programming for finding cost optimal and robust decisions. They are widely used in many branches of engineering like power management under uncertainty. The structural analysis and the algorithmic exploitation of probabilistic constraints heavily relies on convexity properties but has to be prepared to go beyond. The talk considers probabilistic constraints with decisions from a Banach space. It addresses issues like convexity of the feasible set, gradients of probability functions and stability of probabilistic programs under approximations of the underlying distribution.

László Horváth

(Department of Mathematics, University of Pannonia)

Cyclic refinements of the different versions of operator Jensen's inequality

In this talk refinements of the operator Jensen's inequality for convex and operator convex functions are given by using cyclic refinements of the discrete Jensen's inequality. Similar refinements are fairly rare in the literature. Some applications of the results to norm inequalities, to the Hölder-McCarthy inequality and to generalized weighted power means for operators are presented.

Lidia Huerga
(Department of Applied Mathematics, Universidad Nacional de Educación a Distancia)

Characterization of proper efficiency in multiobjective optimization through nonlinear scalarization

(joint work with César Gutiérrez, Vicente Novo)

Without assuming any convexity hypothesis, we characterize proper efficient solutions of a multiobjective optimization problem through nonlinear scalarization. For this aim, we consider that the ordering cone is polyhedral, which lets us derive a manageable characterization, expressed in terms of the matrix that defines the ordering cone. Moreover, in the case when the feasible set is given by a cone constraint, we also obtain necessary and sufficient conditions for proper efficient solutions by means of a nonlinear scalar Lagrangian.

Alfredo N. Iusem

(Instituto de Matemática Pura e Aplicada, Rio de Janeiro)

Second order asymptotic functions

We introduce a new second order asymptotic function which gives information on the convexity (concavity) of the original function from its behaviour at infinity. We establish several properties and calculus rules for this concept, which differs from previous notions of second order asymptotic function. Finally, we apply our new definition in order to obtain necessary and sufficient optimality conditions for quadratic programming and quadratic fractional programming.

Justyna Jarczyk
(Faculty of Mathematics, Computer Science and Econometrics, University of Zielona Góra)

Convex means

(joint work with Dorota Głazowska)

We report some preliminary results of our joint work on convexity of means. At the very beginning we have focused on weighted quasi-arithmetic means studying connections between their convexity and the convexity of their generators. Some examples and counterexamples are presented as well as open problems are posed. Also connections between the convexity of an arbitrary weighted quasi-arithmetic mean generated by a function and that one generated by its inverse are studied.

Witold Jarczyk
(Faculty of Mathematics, Computer Science and Econometrics, University of Zielona Góra)

Convexity in abelian semigroups

(joint work with Zsolt Páles)

We introduce two notions of convexity of sets in abelian semigroups and then suitable notions of convexity for functions defined on such a semigroup. Basic properties of convex sets and functions are examined. We compare various concepts of convex functions, in particular adopting the notion of a convex function defined on a subset of an abelian group, introduced by the speaker and Miklós Laczkovich a few years ago. An outline of a further research is also given.

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Anurag Jayswal
(Department of Applied Mathematics, Indian Institute of Technology,
Indian School of Mines)

Modified objective function approach for multitime variational problems

(joint work with Tadeusz Antczak, Shalini Jha)

The present paper is devoted to study the modified objective function approach used for solving the considered multitime variational problem. In this method, a new multitime variational problem is constructed by modifying the objective function in the original considered multitime variational problem. Further, the equivalence between an optimal solution to the original multitime variational problem and its associated modified problem is established under both hypotheses of invexity and generalized invexity defined for a multitime functional. Thereafter, using the modified objective function method, we derive the saddle-point results for the considered multitime variational problem. Further, we provide some examples to illustrate the results established in the paper.

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Tibor Kiss
(Institute of Mathematics, University of Debrecen)

Implications among generalized convexity properties of real functions

(joint work with Zsolt Páles)

Motivated by the well-known implications among t-convexity properties of real functions, analogous relations among the *upper* and *lower* M*-convexity* properties of real functions will be presented. More precisely, having an n-tuple $(M_1, ..., M_n)$ of continuous two-variable means, the notion of the descendant of these means (which is also an n-tuple $(N_1, ..., N_n)$ of two-variable means) will be introduced. The main results then state that if a function f is convex with respect to the mean M_i for all $i \in \{1, ..., n\}$, then it is also N_i-convex for all $i \in \{1, ..., n\}$.

Fumiaki Kohsaka
(Department of Mathematical Sciences, Tokai University)

Approximating minimizers of convex functions in geodesic metric spaces with curvature bounded above

The proximal point algorithm (PPA, for short) is a well-known method for approximating minimizers of convex functions in Hilbert spaces. Bačák [Israel J. Math. **194** (2013), 689–701] generalized the classical result by Brézis and Lions [Israel J. Math. **29** (1978), 329–345] on this algorithm in Hilbert spaces to that in more general complete CAT(0) spaces. In this talk, using the resolvent introduced by Kimura and Kohsaka [J. Fixed Point Theory Appl. **18** (2016), 93–115], we study the asymptotic behavior of PPA for convex functions in complete CAT(1) spaces.

Dawid Kotrys
(Department of Mathematics, University of Bielsko-Biała)

Separation by Jensen and affine stochastic processes

In 1980 K. Nikodem introduced convex stochastic processes and investigated their regularity properties. In 1992 A. Skowroński obtained some further results on convex stochastic processes which generalize some known properties of convex functions. The aim of this talk is to give conditions under which pairs of stochastic processes can be separated by Jensen and by affine stochastic processes. As a consequence, some stability results of the Hyers-Ulam-type are obtained.

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Alexandru Kristály
(Department of Economics, Babeş–Bolyai University)

Convexity vs. curvature

Various results based on some convexity assumptions (involving the exponential map along with affine maps, geodesics and convex hulls) have been recently established on Hadamard manifolds (simply connected, complete Riemannian manifolds with nonpositive sectional curvature).

In the first part of the talk we prove that these conditions are mutually equivalent and they hold if and only if the Hadamard manifold is isometric to the Euclidean space. In this way, we show that some results in the literature obtained on Hadamard manifolds are actually nothing but their well-known Euclidean counterparts.

In the second part, we use 'right' geodesic convexities in order to guarantee the existence of Nash equilibria on curved spaces. Furthermore, we prove that such an analysis can be successfully developed on Riemannian manifolds with nonpositive curvature, delimiting the Hadamard manifolds as the optimal geometrical framework for Nash-type equilibria.

Felipe Lara
(Departamento de Matemáticas, Universidad de Tarapacá)

Asymptotic analysis for quasiconvex functions with applications

The notion of asymptotic cone of an unbounded set has been introduced in order to study its behavior at infinity. The asymptotic cone of the epigraph of a function, that yields the asymptotic function, provides a description of the function at infinity. These notions are an outstanding tool for studying convex problems with unbounded data and have given rise to the branch of mathematics called asymptotic analysis.

But when dealing with nonconvex functions, the usual notion of asymptotic function does not provide adequate information on the level sets of the original function. The issue of finding a correct definition of asymptotic function in the quasiconvex case was dealt with by Amara, Penot and Flores-Bazán among others.

In this talk, we introduce and develop a new asymptotic function to deal with the class of quasiconvex functions. We provide comparisons with all the previous attempts presented in the literature. Finally, applications to the quadratic fractional programming problem showing the advantage of our new function will be given.

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Maryam Lotfipour
(Department of Mathematics and Applications, University of Fasa)

Convexity conditions of KKM-like theorems via mappings

The convexity condition plays an important role in many problems such as nonempty intersection theorems. There are many topological spaces which do not have the linear structure and the usual convexity condition does not exist in such spaces. Here, we consider various kind of convexity conditions defined by mappings to have KKM-like theorems in topological spaces.

Dinh The Luc
(Department of Mathematics, Avignon University)

On equilibrium in multi-criteria traffic networks

(joint work with Truong Thi Thanh Phuong)

We develop a new method to generate the set of equilibrium flows of a multi-criteria transportation network. To this end we introduce two optimization problems by using a vector version of the Heaviside Step function and the distance function to Pareto minimal elements and show that the optimal solutions of these problems are exactly the equilibria of the network. We study the objective functions by establishing their generic differentiability and local calmness at equilibrium solutions. Then we present an algorithm to generate a discrete representation of equilibrium solutions by using a modified Frank-Wolfe reduced gradient method and prove its convergence. We give some numerical examples to illustrate our algorithm and show its advantage over a popular method by using linear scalarization

Judit Makó

(Department of Analysis, University of Miskolc)

The approximate convexity of the Takagi function

In this talk, I would like to show some interesting property the so-called Takagi function. This function was introduced by T. Takagi in [3] in 1903, and it can be defined by the following way:

$$T(x) = \sum_{n=0}^{\infty} \frac{d_{\mathbb{Z}}\left(2^n x\right)}{2^n} \qquad (x \in \mathbb{R}),$$

where $d_{\mathbb{Z}}(x) := \inf\{|x-z| : z \in \mathbb{Z}\}$ This function plays an important role also in the theory of approximate convexity. The approximate convexity of this function was proved by Z. Boros in [1]. In this talk, I would like to give an another proof of this theorem (see [2]).

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Gyula Maksa
(Department of Analysis, University of Debrecen)

Results related to the invariance equation $K \circ (M, N) = K$

(joint work with Zoltán Daróczy)

Let $I \subset \mathbb{R}$ be an interval of positive length, $K, M, N : I^2 \to I$ be functions. In this talk, we discuss the following problems.

- Suppose that K and M are means in the sense that
$$K(x,y), M(x,y) \in [\min\{x,y\}, \max\{x,y\}] \qquad (x, y \in I),$$
and the invariance equation holds for the functions K, M and N. Find conditions under which the function N will be a mean itself, as well. We present the solution of this problem supposing that K is Matkowski mean, that is,
$$K(x,y) = (f+g)^{-1}(f(x)+g(y)) \qquad (x, y \in I)$$
where $f, g : I \to \mathbb{R}$ are continuous and strictly monotonic functions in the same sense.
- Under given means K and M, what is the solution N of the invariance equation? In special cases, we give the answer to this question, too.

The main motivation of our investigations is the paper

P. Kahlig and J. Matkowski, *Invariant means related to classical weighted means*, Publ. Math. Debrecen, **89/3**(2016), 373–387.

Monica Milasi
(Mathematics and Computer Science, Physical Sciences and Earth Sciences,
University of Messina)

Solutions of quasi-variational inequalities through variational inequalities

A quasi-variational inequality corresponds to a variational inequality in which the constraint set depends on the current value of the variable. The aim of this talk is to identify a class of quasi-variational inequalities for which the existence of solutions is given through auxiliar variational inequalities. Indeed, we introduce two (classical) Stampacchia variational inequality for which existence and reguarity results of solutions are proven. With the help of these results we obtain the existence of solutions for quasi-variational problems.
This class of quasi-variational inequalities is directly inspired by an equilibrium problem for economies involving sequential trade under conditions of uncertainty. A model with these features is represented by the Radner equilibrium. Existence of solutions for this equilibrium problem is then deduced.

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Patriche Monica

(Faculty of Mathematics and Computer Science, University of Bucharest)

New types of correspondences with generalized convexity and their applications to the minimax inequalities

In this talk, we sketch the theory for the generalized convex set-valued maps, which comes from the motivating work on minimax inequalities. This theory encompasses, in particular, the situations when the continuity assumptions are not fulfilled.

Our focus is on introducing several classes of cone convexity. This being the case, we must underline the importance of the behaviour of the set-valued maps $F(\cdot,\cdot) : X\times X \to 2^Y$, in the points where their values contain or not maximal (resp. minimal) elements of a certain set of type $\bigcup_{y\in X} F(x,y)$ or $\bigcup_{x\in X} F(x,y)$. We obtain the new definitions by transferring the convexity properties of the maps from a variable to another, and by taking into consideration the maximal (resp. minimal) elements.

We introduce several classes of set-valued maps with new generalized convexity properties. We also obtain minimax theorems for set-valued maps which satisfy these convexity assumptions and which are not continuous.

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Boris Mordukhovich
(Department of Mathematics, Wayne State University)

Using generalized convexity to avoid slow convergence of primal-dual algorithms in optimization

(joint work with Ebrahim Sarabi)

It has been well recognized that critical multipliers in optimization and variational systems are largely responsible for slow convergence of major primal-dual algorithms involving Lagrange multipliers. The main message of this talk is to present efficient conditions that ensure the absence of critical multipliers associated with stable local minimizers for some classes of variational systems related to generalized convexity. For these purposes, we employ advanced tools of variational analysis and second-order generalized differentiation.

Noémi Nagy
(Institute of Mathematics, University of Miskolc)

Separation theorems for approximately convex functions

(joint work with Zoltán Boros)

In this talk first we present a more abstract version of a result of Baron, Matkowski and Nikodem.

Then we introduce the definitions of $\mathbb{F}$-affine and $\mathbb{F}$-convex functions with respect to a control function sequence (G_n), where $\mathbb{F}$ is a subfield of $\mathbb{R}$, X is a linear space over $\mathbb{F}$, D is an $\mathbb{F}$-convex, $\mathbb{F}$-algebraically open subfield of X, and $G_n : (t_1, t_2, \ldots, t_n) \times D^n \to \mathbb{R}$ is a control function sequence. (Here $n \in \mathbb{N}$ and $t_1, t_2, \ldots, t_n \in [0, 1] \cap \mathbb{F}$ such that $t_1 + t_2 + \ldots + t_n = 1$.)

Finally we examine some properties of these notions and we prove a separation and a stability theorem.

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Kazimierz Nikodem
(Department of Mathematics, University of Bielsko-Biała)

Functions generating $(m, M, \Psi)-$Schur-convex sums

(joint work with Silvestru Sever Dragomir)

The notion of $(m, M, \Psi)-$Schur-convexity is introduced and functions generating $(m, M, \Psi)-$Schur-convex sums are investigated. An extension of the Hardy-Littlewood-Pólya majorization theorem is obtained. A counterpart of the result of Ng stating that a function generates $(m, M, \Psi)-$Schur-convex sums if and only if it is $(m, M, \psi)-$Wright-convex is proved and a characterization of $(m, M, \psi)-$Wright-convex functions is given.

Andrzej Olbryś
(Institute of Mathematics, University of Silesia)

On T-Schur-convex functions

(joint work with Tomasz Szostok)

Motivated by the concept of Schur-convexity we introduce a notion of T-Schur-convexity in the following way: Let T be a fixed $n \times n$ double stochastic matrix, and let D be a convex subset of a real linear space. A function $f : D^n \to \mathbb{R}$ is said to be T-Schur-convex if

$$f(Tx) \leq f(x), \quad x \in D^n.$$

If f is a T-Schur-convex for any double stochastic $n \times n$ matrix T then we say that it is Schur-convex. In our talk we study the properties of T-Schur-convex functions.

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Zsolt Páles
(Institute of Mathematics, University of Debrecen)

Support theorems in an abstract setting

(joint work with Andrzej Olbryś)

We present support theorems for (ω,Ω)-*convex functions* f, i.e., for functions $f : X \to Y$ satisfying the functional inequality

$$f\big(\omega(x_1,\dots,x_n)\big) \le \Omega\big(f(x_1),\dots,f(x_n)\big) \qquad (x_1,\dots,x_n \in X),$$

where we assume that

(1) X is a nonempty set;
(2) $(Y,\le)$ is partially ordered set which is *complete with respect to the infimum of lower bounded chains*;
(3) $\omega : X^n \to X$ is a *reflexive* operation which is also *autodistributive* in each of its variables;
(4) $\Omega : Y^n \to Y$ is a reflexive operation which is also autodistributive and is an *order automorphism* in each of its variables.

Under these conditions, at each ω-*interior point of* X, we prove the existence of an (ω,Ω)-*affine support function* for f.

Cornel Pintea
(Department of Mathematics, Babeş–Bolyai University)

Generalized monotone operators on finite dimensional spaces. Applications

We enlarge the class of Minty-Browder monotone operators to the class of h-monotone operators. The inverse images of continuos Minty-Browder monotone operators on Hilbert spaces are all convex whereas the inverse images of continuous h-monotone operators on the finite dimensional space $\mathbb{R}^n$ are indivisible by closed connected hypersurfaces. The latter property of inverse images is still good enough to ensure the global injectivity of a h-monotone operator when combined with the additional requirement on the operator to be a local homeomorphism. The midle class of δ-monotone operators, which lies in between the class of Minty-Browder monotone operators and the class of h-monotone operators, is a rather important source of h-monotone operators. A δ-monotone operator can be technically detected by means of its parameters of monotonicity, parameters which enjoy some special attention in our approach.

Teresa Rajba
(Department of Mathematics, University of Bielsko-Biała)

On strongly quasi-convex functions

(joint work with Jacek Mrowiec)

Dragomir and Pearce (1998) introduced the class of Jensen-quasi-convex and Wright-quasi-convex functions, as a generalization of the class of quasi-convex functions. We introduce and investigate the notion of strongly quasi-convex, strongly Jensen-quasi-convex and strongly Wright-quasi-convex functions. Let us note, that presented in this paper the notion of strongly quasi-convex function differs from that of Korablev (1980).

We introduce also and study the notion of quasi-convex, Jensen-quasi-convex and Wright-quasi-convex functions of higher order, as well as strongly quasi-convex, strongly Jensen-quasi-convex and strongly Wright-quasi-convex functions of higher order. We give a characterization of quasi-convex and strongly quasi-convex functions of higher order, which is a counterpart of that given for quasi-convex functions.

Juan Luis Ródenas-Pedregosa
(Matemática Aplicada, Universidad Nacional de Educación a Distancia)

Nonlinear scalarization in real linear spaces and vector equilibrium problems

(joint work with César Gutiérrez and Vicente Novo)

In this talk we focus on vector equilibrium problems in which the final space of the bifunction is a real linear space not endowed with any particular topology. We extend the so-called Gerstewitz's functional to this framework via algebraic concepts as the algebraic interior or the vector closure. In particular, its level and sublevel sets are determined and essential properties such as monotonicity and convexity are characterized. It is shown that the obtained results improve some recent ones in the literature. Finally, we apply it to characterize by scalarization the weak efficient solutions of a wide class of vector equilibrium problems whose ordering is defined through a free-disposal set.

Éva Székelyné Radácsi
(Department of Analysis, University of Debrecen)

Characterization of convexity with respect to smooth Chebyshev systems

(joint work with Zsolt Páles)

Popoviciu's well-known theorem states that a real functions f is n-convex if and only if it is $(n-1)$- times continuously differentiable and its $(n-1)$st-order derivative $f^{(n-1)}$ is convex. In 1966, Karlin and Studden managed to generalize the regularity part of this result by proving that if a function f is convex with respest to an n-dimensional sufficiently smooth extended Chebyshev system then it is $(n-2)$-times continuously differentiable. On the other hand, for the convexity part of Popoviciu's theorem, Bessenyei and Páles in 2003 showed that a function f is convex with respect to a 2-dimensional Chebyshev system (ω_1, ω_2) if and only if the function $\frac{f}{\omega_1} \circ \left(\frac{\omega_2}{\omega_1}\right)^{-1}$ is convex in the classical sense.

Motivated by the above two results, for the n-dimensional Chebyshev system setting, it has been an open problem to characterize convexity with respect to the Chebyshev system in terms of classical convexity notions. The aim of this talk is to construct, in terms of the members of the given n-dimensional Chebyshev system, an $(n-2)$nd-order linear differential operator L, such that the convexity of f with respect to the n-dimensional Chebyshev system be equivalent to the convexity of Lf in the classical sense.

Patricia Szokol
(Department of Applied Mathematics and Probability Theory, University of Debrecen)

Transformations preserving norms of means of positive operators

(joint work with Lajos Molnár)

Motivated by recent investigations on norm-additive and spectrally multiplicative maps on various spaces of functions, in this presentation we determine all bijective transformations between the positive cones of standard operator algebras over a Hilbert space which preserve a given symmetric norm of a given mean of elements. (We note that by a standard operator algebra we mean a subalgebra of $B(H)$ the algebra of all bounded linear operators on H which contains all finite rank operators in $B(H)$. Furthermore, we say that the norm N on $B(H)$ is a symmetric norm, if $N(AXB) \leq \|A\| N(X) \|B\|$ holds for all $A, B, X \in B(H)$.)

Tomasz Szostok
(Institute of Mathematics, University of Silesia)

Some applications of Levin-Stechkin theorem and it's generalizations

We show several applications of theorems involving Stieltjes integral to obtain inequalities satisfied by convex functions and convex functions of higher orders. These inequalities are motivated among others by numerical integration and differentiation.

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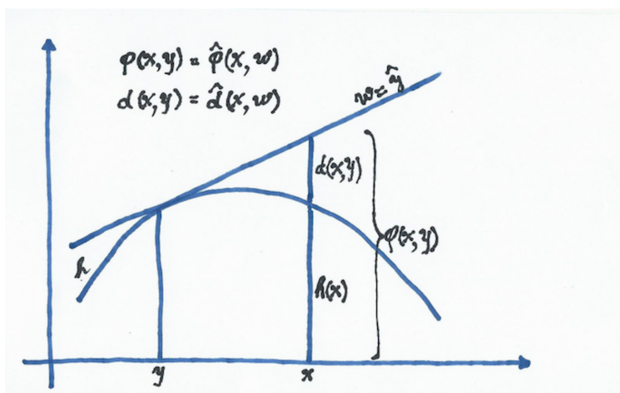
Flemming Topsøe
(Department of Mathematical Sciences, University of Copenhagen)

Models of information and global optimization without Lagrange multipliers



Bregman construction of primitive triple (φ,h,d) from h

Take a look at the figure. Start with the concave function h defined on some interval of the real line. Call it *entropy* and consider the task to find MaxEnt, the maximum entropy value, and the (or a) associated argument. Do not differentiate – that might not tell so much and, in addition, the function needs not be smooth or we may even allow non-concave functions. Instead, seemingly artificially, introduce a two-person zero-sum game by considering points in the interval either as representing the *state of Nature* (x in the figure) or the *belief of Observer* (y in the figure). Think of $h(x)$ as the *effort* needed by Observer in case there is a *perfect match*, i.e. $y = x$. Now then, what is the effort in case the match is not perfect? Thinking about it, $\phi(x, y)$ on the figure is an appropriate answer. And $d(x, y)$ represents the *divergence* or *redundancy*. The straight line $w = \hat{y}$ in the figure represents Observer's *action* in the form of a *control function* (recall what Good said in 1952: *"belief is a tendency to act"*). Now note that if a strategy by Observer is *robust* in the sense that the effort is independent of the state chosen by Nature, you have found what you were looking for, the MaxEnt-value and an associated argument.

Examples: With $h(x) = x \ln \frac{1}{x}$ and by a natural process of integration or just summation, you find key notions related to Shannon's Information Theory, whereas if you take $h(x) = -x^2$, a similar process brings you to classical geometric notions.

The indicated unification of seemingly different areas of research is discussed at length in the author's recent article *Paradigms of Cognition* (*Entropy 2017, doi=10.3390/e19040143*). The indicated result about robustness is what makes it possible to avoid Lagrange multipliers, not a great deed in itself, but it leads you to extra insight.

Csaba Varga

(Faculty of Mathematics, Babeş–Bolyai University)

A Hardy-Brezis-Marcus inequality in Minkowski space with curvature bounds

(joint work with Ioan Radu Peter)

We prove that in a nonreversible Minkowski space the distance function of an open connected set with C^2-boundary is superharmonic in the distribution sense if and only if the boundary is weakly mean convex. We also prove an extension of the Hardy–Brezis–Marcus inequality which in some particular case reduces to a classical Hardy inequality on mean convex domain along the line of Brezis and Marcus. In the so obtained Hardy–Brezis–Marcus inequality the constant depends on the mean curvature of the domain. This result is applied to a minimization problem. The uniformity and the reversibility constant of the Minkowski space play an important role in applications.

Csaba Vincze
(Institute of Mathematics, University of Debrecen)

Lazy orbits: an optimization problem on the sphere

Non-transitive subgroups of the orthogonal group play an important role in the non-Euclidean geometry. If G is a closed subgroup in the orthogonal group such that the orbit of a single Euclidean unit vector does not cover the (Euclidean) unit sphere centered at the origin then there always exists a non-Euclidean Minkowski functional such that the elements of G preserve the Minkowskian length of vectors. In other words the Minkowski geometry is an alternative of the Euclidean geometry for the subgroup G. It is rich of isometries if the group G is "close enough" to the orthogonal group or at least to one of its transitive subgroups. The measure of non-transitivity is related to the Hausdorff distances of the orbits under the elements of G to the Euclidean sphere. Its maximum/minimum belongs to the so-called lazy/busy orbits, i.e. they are the solutions of an optimization problem on the Euclidean sphere. The extremal distances allow us to characterize the reducible/irreducible subgroups. We also formulate an upper and a lower bound for the ratio of the extremal distances.

As another application of the analytic tools we introduce the rank of a group G. We prove that any irreducible subgroup of maximal rank must be finite. Since the reducible and the finite irreducible subgroups form two natural prototypes of subgroups all of whose orbits have a positive measure of non-transitivity, the rank seems to be a fundamental notion in the characterization of non-transitive subgroups.

Finally we present the setting of the application of the results to G as the holonomy group of a metric linear connection on a connected Riemannian manifold.

Peter Volkmann
(Institute of Mathematical Analysis, KIT)

Quasimonotonicity and functional inequalities

(joint work with Gerd Herzog)

A comparison theorem for functional equations in ordered topological vector spaces will be given, which generalizes the results from [1], [2]. Quasimonotonicity is fundamental for these investigations.

References

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Amr Zakaria

(Institute of Mathematics, University of Debrecen and
Department of Mathematics, Ain Shams University)

On the local and global comparison of generalized Bajraktarević means

(joint work with Zsolt Páles)

Given two continuous functions $f, g : I \to \mathbb{R}$ such that g is positive and f/g is strictly monotone, a measurable space $(T, \mathcal{A})$, a measurable family of d-variable means $m : I^d \times T \to I$, and a probability measure μ on the measurable sets $\mathcal{A}$, the d-variable mean $M_{f,g,m;\mu} : I^d \to I$ is defined by

$$M_{f,g,m;\mu}(x_1, \ldots, x_d) := \left(\frac{f}{g}\right)^{-1}\left(\frac{\int_T f\big(m(x_1, \ldots, x_d, t)\big)d\mu(t)}{\int_T g\big(m(x_1, \ldots, x_d, t)\big)d\mu(t)}\right), \qquad (x_1, \ldots, x_d) \in I^d.$$

The aim of this paper is to study the local and global comparison problem of these means, i.e., to find conditions for the generating functions (f, g) and (h, k), for the families of means m and n, and for the measures μ, ν such that the comparison inequality

$$M_{f,g,m;\mu}(x_1, \ldots, x_d) \le M_{h,k,n;\nu}(x_1, \ldots, x_d), \qquad (x_1, \ldots, x_d) \in I^d$$

be satisfied.